Naval Research Laboratory
Washington, DC 20375-5000

NRL Memorandum Report 6597

The Effects of Wiggler Errors on Free Electron Laser Performance

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April 2, 1990

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE 1990 April 2	3. REPORT TYPE AND DATES COVERED Interim

4. TITLE AND SUBTITLE

The Effects of Wiggler Errors on Free Electron Laser Performance

5. FUNDING NUMBERS

6. AUTHOR(S)

Esarey, E., Marable*, W., and Tang, C. M.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Research Laboratory
Washington, DC 20375-5000

8. PERFORMING ORGANIZATION REPORT NUMBER

PR - 9M0214

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

ONR
Arlington, VA 22217

National Insitute of Standards and Technology
Gaithersburg, MD 20899

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NRL Memorandum Report 6597

11. SUPPLEMENTARY NOTES

*University of Maryland, College Park, MD 20742

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release distribution; unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The effects of random wiggler magnetic field errors on free electron lasers is analyzed analytically and computationally. Wiggler field errors perturb the electron beam as it propagates and lead to a random walk of the beam centroid δx as well as cause deviations in the parallel beam energy $\delta\gamma_{\parallel}$ and in the relative phase of the electrons in the ponderomotive wave $\delta\psi$. The phase deviation $\delta\psi$ is identified as the single most important parameter characterizing the detrimental effects of wiggler errors. In order to avoid significant reduction in gain it is necessary for the phase deviation to be small compared to π. It is shown that transverse focusing of the electron beam is not effective in reducing the phase deviation (i.e., transverse focusing reduces the average phase deviation by 1/2). Furthermore, it is shown that the results of beam steering at the wiggler entrance reduces the average phase deviation at the end of the wiggler by 1/3. The detrimental effects of wiggler errors may be reduced by arranging the magent poles in an optimal ordering such that the magnitude of the phase deviation is minimized.

14. SUBJECT TERMS	15. NUMBER OF PAGES
Free electron lasers Random field errors	26
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SR

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

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THE EFFECTS OF WIGGLER ERRORS ON FREE ELECTRON LASER PERFORMANCE

1. Introduction

Intrinsic magnetic field errors δB are present in any realistic wiggler magnet. Such errors are unavoidable and arise from imperfections in the fabrication and assembly of wiggler magnets. State-of-the-art wiggler construction techniques yield rms field errors on the order[1] $(\delta B/B_w)_{rms} \simeq 0.1 - 0.5\%$. These field errors perturb the electron beam as it propagates through the wiggler and lead to i) a random walk of the beam centroid,[2-4] δx, ii) variations in the parallel beam energy,[5-8] $\delta\gamma_{\parallel}$, and iii) variations in the relative phase of the electrons in the ponderomotive potential,[5-8] $\delta\psi$. If left uncorrected, field errors ultimately decrease free electron laser (FEL) gain[2-8] (this reduction becomes more significant for long wigglers). Reduction in gain may occur from a loss of optical guiding (due to large δx) or from a loss of FEL resonance (due to large $\delta\psi$).

Past research, for the most part, has been primarily concerned with the random walk δx. It has been shown that the random walk δx may be effectively controlled by i) transverse beam focusing[3-8] (finite k_β, where k_β is the betatron wavenumber) and by ii) periodic beam steering.[2-8] By using either one or a combination of beam focusing and periodic steering, in principle, the random walk δx may be kept as small as desired. The major conclusions of the present work are the following. Given that the random walk δx may be effectively controlled, the phase deviation $\delta\psi$ may be identified as the single most important parameter characterizing the effects of wiggler errors.[5-8] In particular, in order to avoid significant reduction in gain, it is necessary that $|\delta\psi| < \pi$. In addition, transverse beam focusing is not effective in controlling $\delta\psi$. Specifically, it may be shown that at the wiggler end $\langle\delta\psi\rangle = (1/2)\langle\psi(k_\beta = 0)\rangle$, where $\langle\rangle$ signifies an ensemble average. Furthermore, beam steering may be used to reduce $|\delta\psi|$ when[8] $L_S < \lambda_\beta$, where L_S is the length over which the steering performed and $\lambda_\beta = 2\pi/k_\beta$. As an example, for $k_\beta = 0$ and one steering segment, $\langle\delta\psi\rangle = (1/3)\langle\delta\psi\rangle_N$, where $\langle\delta\psi\rangle_N$ is the value in the absence of steering.

As a further motivation, it is appropriate to consider some aspects of wiggler design. Typically, when "ordering" a wiggler from a vendor, limits are placed on δB_{rms} and $|\int dz\delta B|$. To meet these specifications, the vendor may arrange the magnet pole in an optimum sequence such that $|\int dz\delta B|$ is minimized. The present research indicates, however, that the optimum "figure of merit" to minimize is not the line integral $|\int dz\delta B|$, but the magnitude of the phase deviation $|\delta\psi|$.

Manuscript approved November 30, 1989.

2. Random Walk of the Beam Centroid

As the electron beam propagates through the wiggler, the electrons experience random velocity kicks $\delta v_\perp$ via the $v_\parallel \times \delta B_\perp$ random force. The equation of motion for the electron beam centroid motion including transverse gradients (weak focusing) is given by[6] $d^2\delta x/dz^2 = -k_\beta^2 \delta x + k_w a_w \delta \hat{B}_y/\gamma$, where k_w is the wiggler wavenumber, k_β is the betatron wavenumber ($k_\beta = k_w a_w/(\sqrt{2}\gamma)$ for a helical wiggler), $\delta\hat{B} = \delta B/B_w$, B_w is the ideal wiggler peak magnetic field, $a_w = eB_w/k_w mc^2$, γ is the relativistic factor of the electron beam and z is the axial propagation distance. This equation may be solved to give the random centroid motion[6]

$$\delta\beta_x = \frac{a_w k_w}{\gamma}\int_0^z dz' \cos k_\beta(z'-z)\delta\hat{B}_y(z') \tag{1a}$$

$$\delta x = -\frac{a_w k_w}{\gamma k_\beta}\int_0^z dz' \sin k_\beta(z'-z)\delta\hat{B}_y(z'), \tag{1b}$$

where $\delta\beta = \delta v/c$.

Given the precise functional dependence of the wiggler errors $\delta B(z)$ for a given wiggler, the above expressions may be used to calculate $\delta x(z)$ for that specific wiggler. However, one does not always know ahead of time the full functional dependence of $\delta B(z)$. Instead, one may know only certain statistical properties of the field errors, such as the rms value δB_{rms}. Hence, it is useful to consider an ensemble of statistically identical wigglers for which the statistical properties of the field errors are known. By performing appropriate averages over this ensemble, one may determine the mean $\langle Q\rangle$ and variance σ for a quantity Q and, hence, determine the most probable range of a single realization of Q. Throughout the following, $\delta B(z)$ is assumed[6] to be a random function with zero mean, finite variance and with an autocorrelation distance given by $z_{ac} \simeq \lambda_w/2$. Also, in the following, a helical wiggler will be assumed and generalization of the results for a linear wiggler is straightforward.[6]

Statistically averaging over an ensemble of wigglers, it is possible to determine the mean-square centroid motion[6]

$$\langle\delta\beta_x^2\rangle = D\left(z + \frac{\sin 2k_\beta z}{2k_\beta}\right) \tag{2a}$$

$$\langle\delta x^2\rangle = \frac{D}{k_\beta^2}\left(z - \frac{\sin 2k_\beta z}{2k_\beta}\right), \tag{2b}$$

where $D = a_w^2 k_w^2 \langle\delta\hat{B}_y^2\rangle z_{ac}/(2\gamma^2)$. Physically, the centroid orbits δx and $\delta\beta_x$ represent diffusing betatron orbits characterized by a diffusion coefficient D. Notice that by increasing

k_β^2 by additional external focusing, one may, in principle, keep δx_{rms} as small as desired. Furthermore, notice that in the 1D limit, $(2k_\beta z)^2 << 1$, $\langle \delta\beta_x^2 \rangle = 2Dz$ and $\langle \delta x^2 \rangle = 2Dz^3/3$. Hence, weak focusing (finite k_β) is effective in reducing the asymptotic scaling of the random walk δx_{rms} from $z^{3/2}$ to $z^{1/2}$. To avoid loss of optical guiding it is desirable to keep $\langle \delta x^2 \rangle << r_s^2$, where r_s is the radiation spot size.

3. Variations in the Parallel Beam Energy

Not only do the field errors perturb the perpendicular motion of the electrons, they also perturb the parallel motion. This is true since a static magnetic field conserves total electron energy. The parallel motion may easily be calculated[6] using the above expressions for the perpendicular motion along with $\beta_\parallel^2 + \beta_\perp^2$ =constant. One may calculate various statistical moments of the parallel motion,[6] such as the mean parallel energy variation $\langle \delta\gamma_\parallel \rangle = \langle \gamma_\parallel \rangle - \gamma_{\parallel 0}$,

$$\frac{\langle \delta\gamma_\parallel \rangle}{\gamma_{\parallel 0}} = -\frac{(1 + a_w^2/4)}{(1 + a_w^2)^2} a_w^2 k_w^2 \langle \delta \hat{B}^2 \rangle z_{ac} z, \tag{3}$$

where the limit $(2k_\beta z)^2 >> 1$ has been assumed.

Statistically, $\langle \delta\gamma_\parallel \rangle$ may be interpreted as an effective energy spread due to field errors.[6] This effective energy spread may lead to a loss of FEL resonance. Heuristically, in order to maintain resonance, one expects that in the low or high gain regime the effective energy spread must be small compared to the intrinsic FEL efficiency η, $|\langle \delta\gamma_\parallel \rangle|/\gamma_{\parallel 0} < \eta$. In the trapped particle regime, maintaining resonance implies that the effective energy spread must be small compared to the depth of the ponderomotive well, $|\langle \delta\gamma_\parallel \rangle|/\gamma_{\parallel 0} < |e\Phi_p|/(\gamma mc^2)$, where Φ_p is the ponderomotive potential. For example, in the low gain regime, $\eta = 1/(2N)$. The inequality $|\langle \delta\gamma_\parallel \rangle|/\gamma_{\parallel 0} < \eta$ implies $\delta \hat{B}_{rms} < 1/(\pi N) \simeq 0.3\%$ for $N = 100$ (where $a_w^2 >> 1$ has been assumed).

4. Deviations in the Relative Phase

To quantify how the parallel energy variation affects FEL gain, it is necessary to consider the relative phase ψ of the electrons in the ponderomotive wave, $d\psi/dz \equiv k + k_w - \omega/(c\beta_z)$. In the small signal limit ($a_R \to 0$, where a_R is the normalized radiation field), the deviation in phase $\delta\psi$ due to the field errors is given by

$$\delta\psi = -\frac{\omega}{2c} \int_0^z dz'(2\beta_{\perp 0}\delta\beta_\perp + \delta\beta_\perp^2), \tag{4}$$

where $\beta_{\perp 0}$ is the ideal wiggle motion (in the absence of field errors) and where $\delta\beta_\perp$ is given by Eq. (1a). Statistically averaging over the wiggler ensemble gives

$$\langle\delta\psi\rangle = -\frac{a_w^2 k_w^3}{(1+a_w^2)}\langle\delta\bar{B}^2\rangle\frac{z_{ac}}{2}\left[\frac{z^2}{2} + \frac{1}{4k_\beta^2}(1-\cos 2k_\beta z)\right]. \tag{5}$$

Notice that $\langle\delta\psi\rangle \simeq C_0 z^2/2$ in the limit $(2k_\beta z)^2 >> 1$; and $\langle\delta\psi\rangle \simeq C_0 z^2$ in the limit $(2k_\beta z)^2 << 1$. Hence, transverse focusing only reduces $\langle\delta\psi\rangle$ by $1/2$. It should be mentioned that in the trapped particle regime, the effects of the synchrotron motion of the electrons may further reduce[4] $\langle\delta\psi\rangle$.

Physically, $\delta\psi$ may be interpreted as an oscillation of the ponderomotive well due to field errors. Maintaining FEL resonance requires $\delta\psi$ to be small compared to π, i.e., the width of the well. In the low gain regime, this phase deviation must be kept small over the entire wiggler length L. Requiring $|\langle\delta\psi(z=L)\rangle| < \pi$ implies $\delta\bar{B}_{rms} < 1/(\pi N) \sim 0.3\%$ for $N = 100$ (where $a_w^2 >> 1$ has been assumed). This is the same condition as obtained above from considering the effective energy spread. In the high gain regime, the situation is somewhat different, since the length scale over which the FEL resonant interaction occurs is the e-folding length $1/\Gamma$, where Γ is the spatial growth rate of the radiation. Maintaining resonance in the high gain regime corresponds to keeping $\delta\psi$ small over an e-folding length: $|\langle\delta\psi(z=1/\Gamma)\rangle| < \pi$. Since, typically $1/\Gamma << L$, one expects the high gain not to be strongly affected by the phase deviation $\delta\psi$ (in contrast to the low gain).

5. Degradation of FEL Gain

Quantitatively, the effect of the phase deviation on the FEL gain in the low gain regime may be determined analytically. The normalized mean amplitude gain is related to $\delta\psi$ by the following expression,

$$\langle\hat{G}\rangle = \int_0^z dz' \int_0^{z'} dz''(z'-z'')\langle\sin[\mu k_w(z'-z'') + \Delta\delta\psi]\rangle, \tag{6}$$

where $\Delta\delta\psi \equiv \delta\psi(z') - \delta\psi(z'')$ and μ = normalized frequency mismatch. Setting $\Delta\delta\psi = 0$ in the above equation gives the gain in the absence of field errors.

Evaluation of the ensemble average in the above expression is dependent on the statistical distribution of the function $\Delta\delta\psi$. Recall that the phase deviation $\delta\psi$ is proportional to terms which are linear in the field error δB as well as terms which are quadratic in the field error, as indicated by Eq. (4). If the field error δB is a Gaussian distributed

random variable, then terms quadratic in δB tend to obey a Gamma distribution. Hence, if the quadratic terms dominate in the expression for $\delta\psi$, then $\delta\psi$ will tend to be Gamma distributed. Assuming $\Delta\delta\psi$ to be approximately Gamma distributed allows the ensemble average in Eq. (6) to be evaluated using the Rice–Mandel approximation,[2,9] yielding

$$\langle \hat{G} \rangle = \int_0^z dz' \int_0^{z'} dz''(z' - z'') \left(1 + \langle \Delta\delta\psi \rangle^2 / f^2\right)^{-f/2} \times \sin\left[\mu k_w (z' - z'') + f \tan^{-1}\left(\langle \Delta\delta\psi \rangle / f\right)\right], \tag{7}$$

where $f = \langle \Delta\delta\psi \rangle^2 / \left(\langle \Delta\delta\psi^2 \rangle - \langle \Delta\delta\psi \rangle^2\right)$.

It is possible to show that the mean gain is a function of only two parameters, $\langle \hat{G} \rangle = F(\mu, \langle \delta\psi \rangle_{max})$, where $\langle \delta\psi \rangle_{max} = \langle \delta\psi(z = L) \rangle$. Furthermore, one can show that $\langle \hat{G} \rangle$ decreases as $\langle \delta\psi \rangle_{max}$ increases. In a similar fashion, it is possible to calculate expressions for the variance of the gain. This variance tends to be large, as is indicated by the numerical simulations discussed below.

Equation (7) may be evaluated numerically to determine the behavior of the mean gain. Figure 1 illustrates this behavior, in which the mean gain $\langle \hat{G} \rangle$ is plotted as a function of the frequency mismatch μ for several values of rms field error $\delta\hat{B}_{rms}$. The parameters in Fig. 1 correspond to a linearly polarized wiggler with $B_w = 5.4$ kG, $\lambda_w = 2.8$ cm, $L = 3.6$ m and $\gamma = 350$ in the limit $k_\beta = 0$ (transverse focusing is neglected). Figure 2 shows the peak gain $\langle \hat{G} \rangle_{max}$ as a function of rms field error $\delta\hat{B}_{rms}$. In Fig. 2, the solid line shows the solution to Eq. (6) in which the ensemble average is evaluated numerically (assuming a uniform distribution of field errors between $\pm\delta B_{max}$), whereas the dashed curve shows the solution to Eq. (7) in which the ensemble average is evaluated using the Rice–Mandel approximation. The circles in Fig. 2 are the result of an FEL simulation code for individual wiggler realizations (particular arrangements of random field errors). In these simulation runs, a random field error model similar to that of Kincaid[2,6–8] was used along with an electron beam of current 2.0 A with an emittance of 10 μm-rad. Notice that the large spread in the simulation results indicates a relatively large variance of the gain.

It is also possible to calculate the effect of wiggler errors on the spatial growth rate in the high gain regime.[7] Figure 3 shows the numerically evaluated spatial growth rate Γ (normalized to the value in the absence of field errors) as a function of the rms field error $\delta\hat{B}_{rms}$. In Fig. 3, the solid points indicate the mean growth rate and the error bars indicate one standard deviation about that mean. These results are for a linearly polarized wiggler with $B_w = 2.4$ kG, $\lambda_w = 8.0$ cm and $L = 15$ m; and for an electron beam of energy 50 MeV with a current of 1.5 kA and an emittance of 4.4 μm-rad. Notice that even for large

rms field errors, $\delta\hat{B}_{rms} = 0.5\%$, the mean spatial growth rate is only slightly reduced (by $< 4\%$). This is in agreement with the discussion presented in the previous section.

6. Beam Steering

One method for reducing the detrimental effects of field errors is through the use of beam steering[2-8] (external fields are used to steer the electron beam back to axis). Analytically, this may be modeled by injecting the electron beam with an initial perpendicular velocity $\beta_{\perp 0}$ such that the centroid displacement is zero at the end of the wiggler $\delta x(z = L) = 0$. The intial perpendicular velocity may be specified in terms of the perturbed perpendicular velocity in the absence of steering $\delta\beta_{\perp N}$ by the relation

$$\beta_{\perp 0} = -\frac{1}{L}\int_0^L dz' \delta\beta_{\perp N}(z'), \qquad (8)$$

where $\delta B_{\perp N}$ is given by Eq. (1a).

Using the above expression for $\beta_{\perp 0}$, one may calculate the electron motion in the presence of the field errors including the effects of beam steering. For example, the phase deviation in the absence of transverse focusing ($k_\beta = 0$) is given by

$$\langle\delta\psi\rangle = -\gamma_{\parallel 0}^2 k_w D\left[z^2 + \epsilon\frac{2}{3}zL\left(1 - \frac{3z}{L} + \frac{z^2}{L^2}\right)\right], \qquad (9)$$

where $\epsilon = 1$ with steering and 0 without steering and where $\gamma_{\parallel 0}$ is the parallel relativistic factor in the absence of field errors. In particular, notice that the effect of steering is to reduce the mean phase deviation by a factor of 1/3, $\langle\delta\psi(L, \epsilon = 1)\rangle = (1/3)\langle\delta\psi(L, \epsilon = 0)\rangle$. It is also possible to calculate $\langle\hat{G}\rangle$ including the effects of steering.

The effect of beam steering at the wiggler entrance on the phase deviation $\delta\psi$ is illustrated in Fig. 4 for the cases (a) without steering and (b) with steering. Here the solid curves represent the mean $\langle\delta\psi\rangle$ and the dashed curves represent one standard deviation about the mean $\langle\delta\psi\rangle \pm \sigma$, where σ is the variance of the phase deviation. These plots are for a linearly polarized wiggler with $B_w = 5.4$ kG, $\lambda_w = 2.8$ cm, $L = 3.6$ m, $\gamma = 350$ and $\delta\hat{B}_{rms} = 0.3\%$ in the limit $k_\beta = 0$ (transverse focusing is neglected). Notice that the effect of steering at the wiggler entrance reduces $\langle\delta\psi\rangle$ by 1/3 at the end of the wiggler, as is indicated by Eq. (9). Also, notice that steering has reduced the variance of the phase deviation by an equally significant amount. For cases in which $k_\beta \neq 0$, it is possible to show[8] that steering reduces the mean phase deviation when the length over which the

steering in performed is less than the betatron wavelength, $L_s < \lambda_\beta$. For cases in which $L_s > \lambda_\beta$, beam steering may increase the value of $\langle\delta\psi\rangle$.

The effect of beam steering at the wiggler entrance on the FEL gain (in the low gain regime) is illustrated in Fig. 5. Here the peak normalized gain $\langle\hat{G}\rangle_{max}$ is plotted as a function of the rms field error $\delta\hat{B}_{rms}$ for the case with steering and for the case with no steering. The parameters in Fig. 5 correspond to a linearly polarized wiggler with $B_w = 5.4$ kG, $\lambda_w = 2.8$ cm, $L = 1.8$ m and $\gamma = 350$ in the limit $k_\beta = 0$. Figure 5 indicates that the mean gain may be significantly enhanced by using steering at the wiggler entrance.

7. Error Reduction Techniques

Several methods exist for reducing the detrimental effects of wiggler errors. Above it was discussed how steering[2-8] the electron beam at the entrance of the wiggler may improve FEL performance. This concept may be generalized to the case of multiple beam steering,[3,4,8] in which the electron beam is steered back to axis in several places along the length of the wiggler. In addition to beam steering, one may consider wiggler errors which are correlated.[8] The results discussed above are for wigglers with random errors which are assumed to be uncorrelated for separation distances greater than $z_{ac} \simeq \lambda_w/2$. By considering a wiggler in which the error for a given magnet pole is correlated to the errors of the surrounding poles, one may construct beneficial correlations which reduce the detrimental effects of the errors.

Alternatively, one may reduce the detrimental effects of the errors by considering an optimal arrangement of the magnet poles.[10-12] That is, the magnet poles are to be arranged in such a way that the detrimental effects of the error of a given pole tend to cancel those of the surrounding poles. More specifically, the magnet poles are arranged in such a way as to minimize an appropriate "cost function". For example, one may choose to arrange the poles such that the magnitude of random walk $|\delta x|$ is minimized, where $\delta x \sim \int dz' \sin k_\beta(z'-z)\delta\hat{B}_y(z')$. (Notice that minimization of $|\int dz\delta B|$ does not correspond to minimization of $|\delta x|$.) However, the results discussed above indicate that a more appropriate cost function is the magnitude of the phase deviation $|\delta\psi|$, $\delta\psi \sim \int dz'(2\beta_{\perp 0}\delta\beta_\perp + \delta\beta_\perp^2)$, where $\delta\beta_x \sim \int dz' \cos k_\beta(z'-z)\delta\hat{B}_y(z')$. By minimizing $|\delta\psi|$, one reduces the amount of gain loss. Ideally, one would like to maximize the actual expression for the gain, Eq. (6), but the functional dependence of the gain on the field errors appears much too complicated to be of practical usefulness.

8. Conclusions

The analytical and numerical work discussed above indicates that the the phase deviation $\delta\psi$ is the single most important parameter characterizing the effects of wiggler errors. Although transverse beam focusing and beam steering are highly effective in controlling the random walk δx (in principle, δx may be kept as small as desired), this is not the case for the phase deviation $\delta\psi$. Transverse beam focusing only reduces the mean phase deviation by a factor of 1/2, $\langle\delta\psi\rangle = (1/2)\langle\psi(k_\beta = 0)\rangle$. Beam steering may be used to reduce $|\delta\psi|$ only when $L_S < \lambda_\beta$. As an example, for the case $k_\beta = 0$ and using steering at the wiggler entrance indicates that the mean phase deviation at the wiggler end is reduced by a factor of 1/3, $\langle\delta\psi(\epsilon = 1)\rangle = (1/3)\langle\delta\psi(\epsilon = 0)\rangle$. The phase deviation leads to a reduction of FEL gain (the low gain regime is affected more strongly than the high gain regime). To avoid significant loss in gain, the above analysis implies that $|\langle\delta\psi\rangle| < \pi$. In the low gain regime, this gives

$$\delta\hat{B}_{rms} < \alpha/(\pi N), \tag{10}$$

where $\alpha = (1 + a_w^2)^{1/2}/a_w$ for a helical wiggler. Possible error reduction techniques include multiple beam steering, correlation of field errors and optimal arrangement of magnet poles. An optimal arrangement of poles corresponds to minimization of $|\delta\psi|$, where $\delta\psi$ is given by Eq. (4).

Acknowledgements

The authors would like to acknowledge useful discussions with P. Sprangle. This work was supported by the Office of Naval Research, Contract No. N00014-87-f-0066, through the National Institute of Standards and Technology; and by the U.S. Department of Energy.

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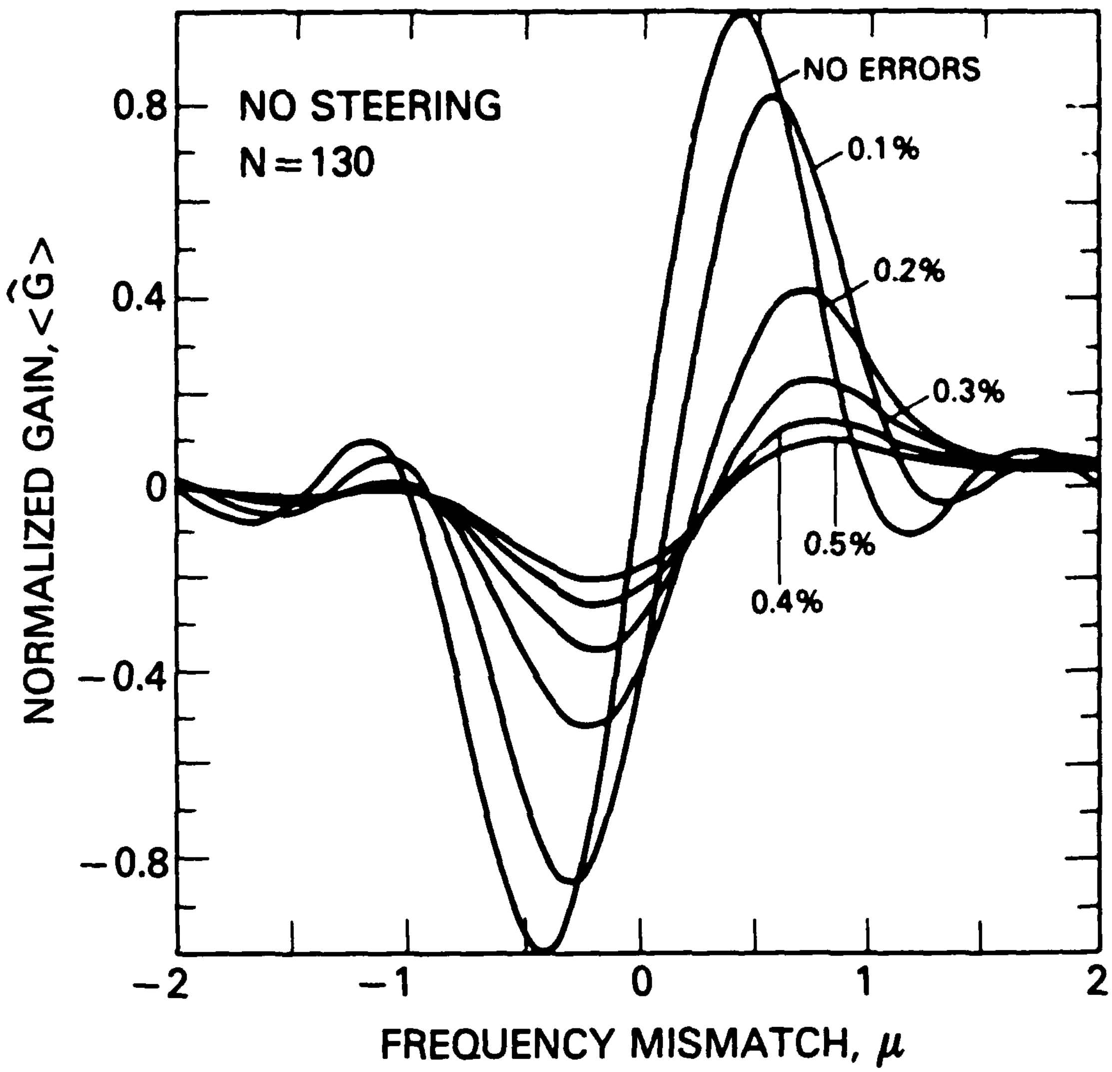


Fig. 1. Mean gain $\langle \hat{G} \rangle$ versus frequency mismatch μ for several values of rms field error $\delta \hat{B}_{rms}$ for a linearly polarized wiggler with B_w = 5.4 kG, λ_w = 2.8 cm, L = 3.6 m and $\gamma = 350$ in the limit $k_\beta = 0$.

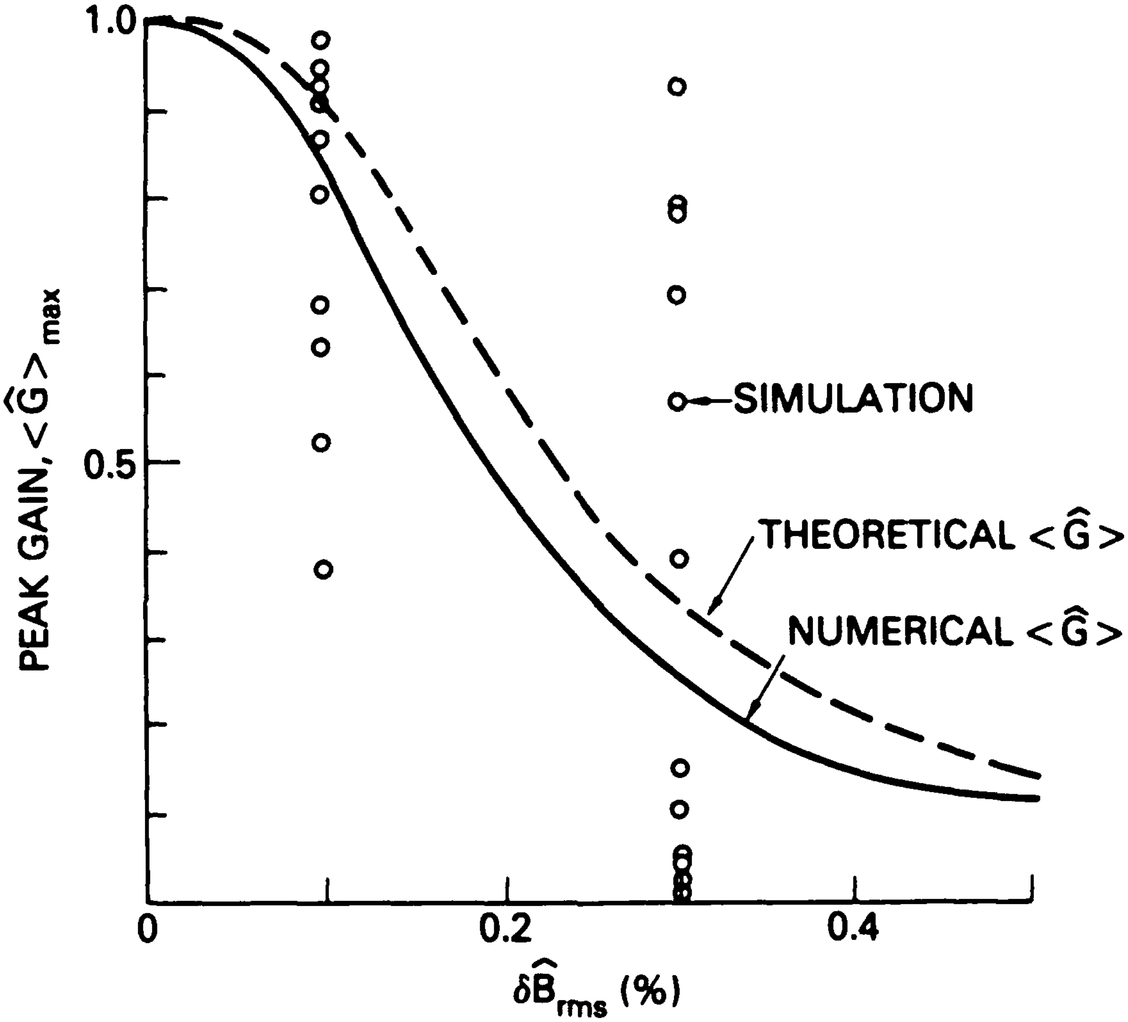


Fig. 2. Peak mean gain $\langle \hat{G} \rangle_{max}$ versus rms field error $\delta \hat{B}_{rms}$ for a linearly polarized wiggler with $B_w = 5.4$ kG, $\lambda_w = 2.8$ cm, $L = 3.6$ m and $\gamma = 350$ in the limit $k_\beta = 0$. The solid curve denotes a numerical average, the dashed curve denotes a theoretical average and the circles denote FEL simulations.

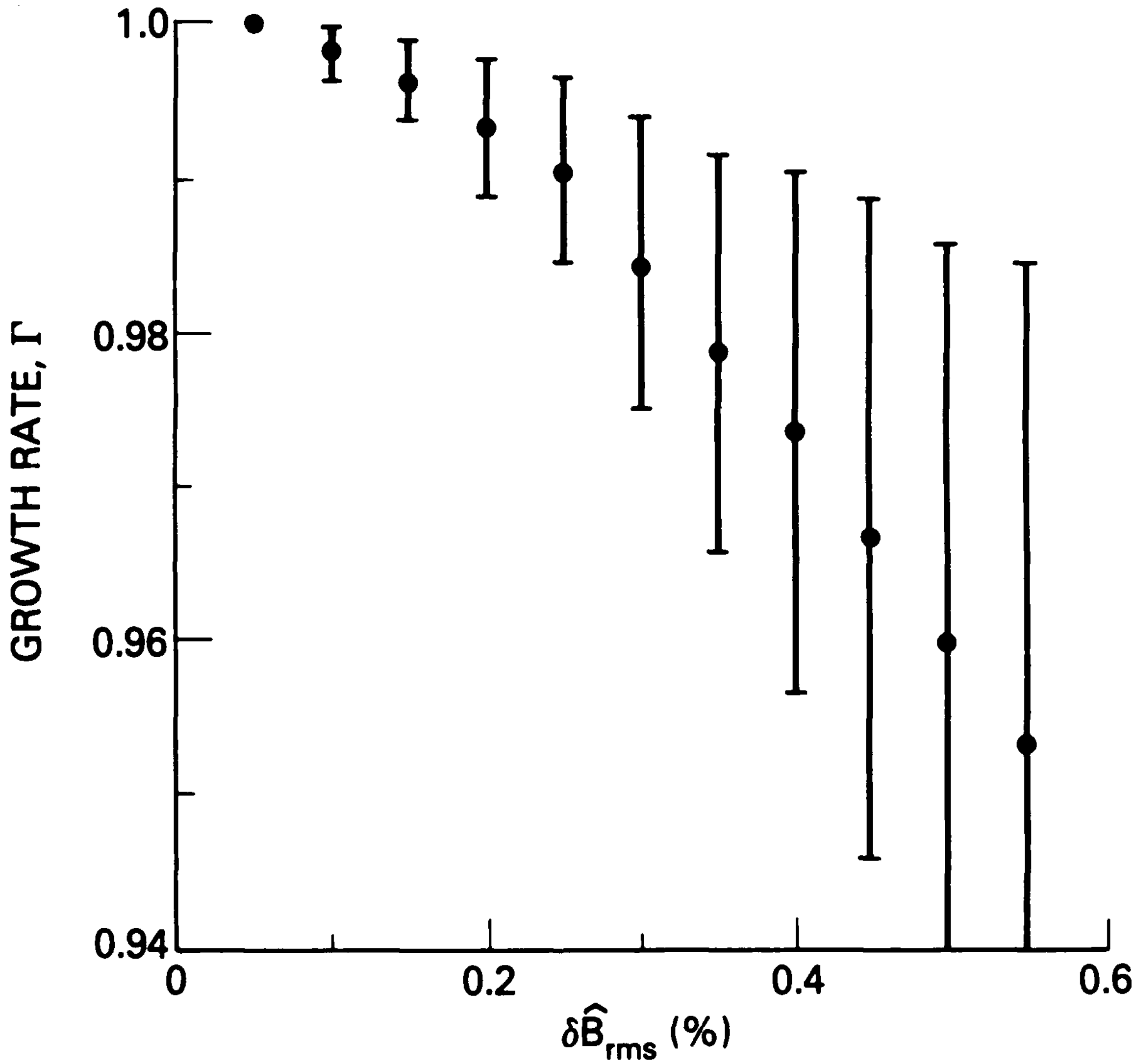
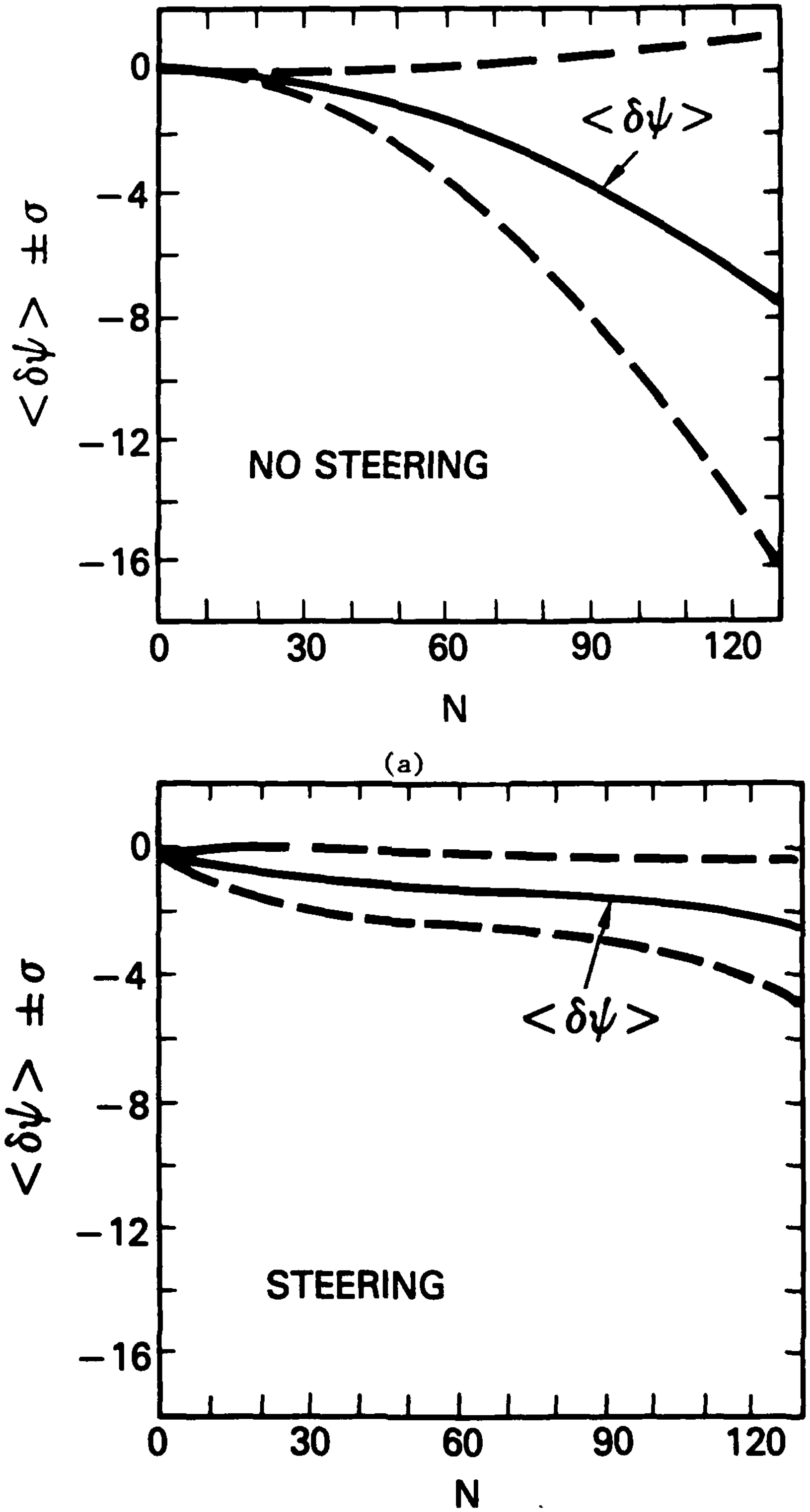


Fig. 3. High gain spatial growth rate Γ versus rms field error $\delta\hat{B}_{rms}$ for a linearly polarized wiggler with $B_w = 2.4$ kG, $\lambda_w = 8.0$ cm and $L = 15$ m and $\gamma = 100$. The solid points denote the mean and the error bars denote one standard deviation.



(a)

(b)

Fig. 4. Phase deviation $\delta\psi$ versus number of wiggler periods N (a) without steering and (b) with steering for a linearly polarized wiggler with $B_w = 5.4$ kG, $\lambda_w = 2.8$ cm, $L = 3.6$ m, $\gamma = 350$ and $\delta\hat{B}_{rms} = 0.3\%$ in the limit $k_\beta = 0$. The solid curves represent the mean $\langle\delta\psi\rangle$ and the dashed curves represent one standard deviation σ about the mean.

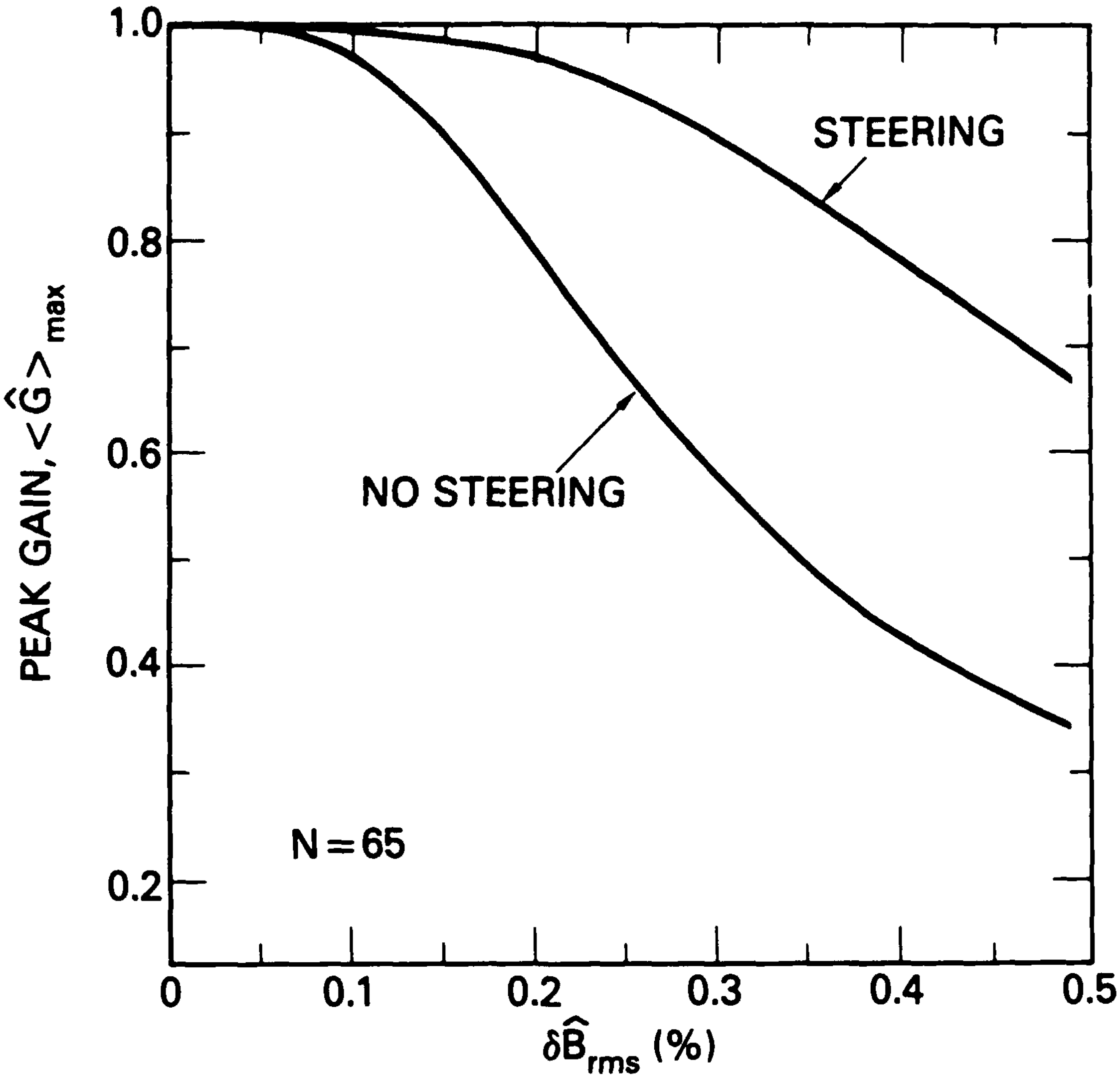


Fig. 5. Peak gain $\langle \hat{G} \rangle_{max}$ versus rms field error $\delta \hat{B}_{rms}$ with and without steering for a linearly polarized wiggler with $B_w = 5.4$ kG, $\lambda_w = 2.8$ cm, $L = 1.8$ m and $\gamma = 350$ in the limit $k_\beta = 0$.

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